

## Thermodynamics

## Heat and Mass Transfer

- |   |  |
|---|--|
| <ul style="list-style-type: none"> <li>• It deals with the equilibrium of the matter</li> <li>• It does not include temperature gradient <math>(\frac{dt}{dx})</math>.</li> <li>• When a System Changes from one equilibrium state to another, it helps to determine the quantity of work and heat interaction</li> </ul> <p>Ex- When a hot rod is placed in water then thermodynamics may be used to predict the final equilibrium temp. of hot rod and water combination.</p> | <ul style="list-style-type: none"> <li>• It is inherently a non-equilibrium process -</li> <li>• It includes temp. gradient <math>(\frac{dt}{dx})</math>.</li> <li>• When a System Changes from one equilibrium state to another, it helps to determine the rate at which energy is transferred.</li> </ul> <p>Ex- When a hot rod is placed in water, then heat transfer may be used to predict the temp. of both the rod and water as a function of time.</p> |
|---|--|

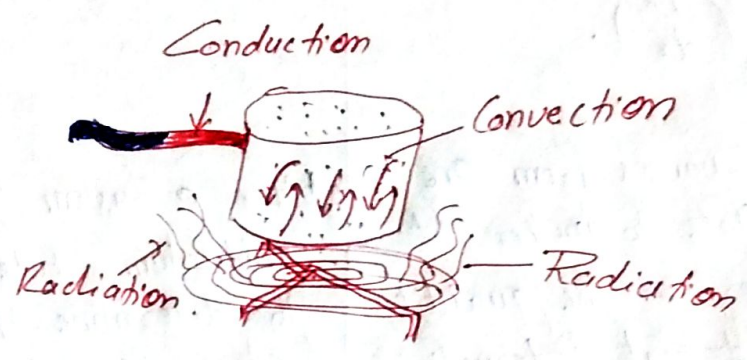
## Modes of Heat Transfer

1. Conduction.
2. Convection.
3. Radiation.

### • Conduction |

Conduction is the transfer of heat from one part of substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.

- In Solids, heat is conducted by lattice vibration and transport of free electrons.
  - In gas, heat is transferred by momentum transfer.
  - In liquids, heat is also transferred by moment transfer.
- Ex:- In metal rod heat is transferred by conduction.



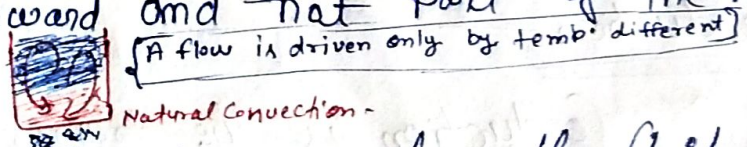
# Convection :-

"Convection is the transfer of heat within a fluid by mixing of one portion of the fluid with another."

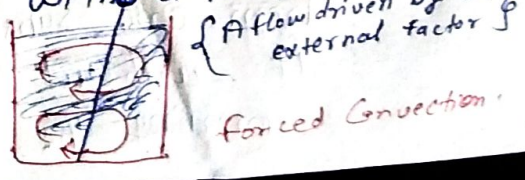
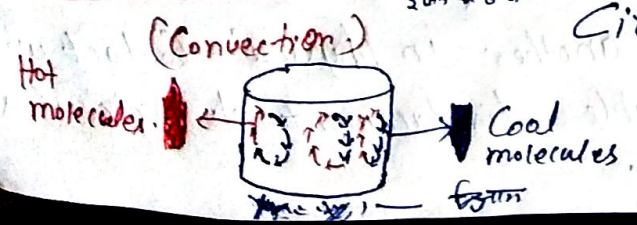
Ex - Water is heated by the convection method.

Convection is a 2 type

• Natural Convection :- It occurs when the fluid circulates by virtue of differences in densities of hot and cold fluid. Cold part of the fluid moves downward and hot part of the fluid goes up.



• Forced Convection :- It occurs when the fluid circulates with a pump. {A flow driven by an external factor}

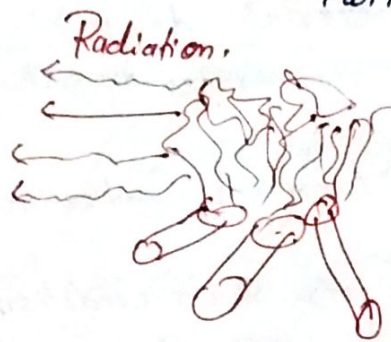


## Radiation !

(2)

"Radiation" is the transfer of Heat through Space or matter by means other than Conduction or Convection.

- Ex - (i) - A burning Candle emits radiation in the form of Heat and light.  
(ii) - The Sun emits radiation in the form of light, Heat and Particles.



## # Fourier's law of Heat Conduction †

"According to Fourier's law, Rate of Heat transfer is directly proportional to Cross-sectional area of the body and also proportional to temp. gradient.

$$Q \propto A \cdot \frac{dt}{dx}$$

(Unit - watt)

$$Q = -KA \frac{dt}{dx}$$

Where -

K = Thermal Conductivity of the body

A = Cross-sectional area of the body.

dt = Difference in temp.

dx = Thickness of the body in direction of flow

[ -ve Sign indicates decrease in temp. along with increase in thickness ]

## • Assumptions for Fourier's law -

1. The heat flow is unidirectional.
2. There is no internal heat generation.
3. The material is homogeneous and isotropic.
4. The temp. gradient is constant.
5. Heat flow takes place under steady state condition.

- Important term -

Isotropic: - Properties of material do not depend on surface orientation (same properties in all directions).

Homogeneous - The material has uniform composition. (S)

Steady State Condition - Under steady state condition the temp. within the system does not change with time.

## # Unit of thermal conductivity: -

We know that the rate of heat transfer -

$$Q = -k \cdot A \frac{dt}{dx}$$

$$k = \frac{Q \cdot dx}{m^2 \cdot A \cdot dt} \text{ - m.k.e}$$

$$\text{Unit of } k = \frac{W}{m^2} \cdot \frac{m}{K}$$

$$k = W / (m \cdot K) \text{ Kelvin}$$

## Effect of Various Parameters on Thermal Conductivity ③

- A) Temperature :-
- In Solid, thermal conductivity decreases with increase in temp.  $t$
  - In liquid thermal conductivity decreases with increase in temp.
  - In gases thermal conductivity increases with increase in temp.

B) Density :- Thermal conductivity increases with increase in density.

C) Chemical Composition - It is highest for pure metals and decreases with increase in impurity.

d) Pressure - Thermal conductivity is weakly dependent on pressure.

E) Dampness - Conductivity for damp material is considerably higher than of dry material.

## # Thermal Resistance

As per Ohm's law - we have

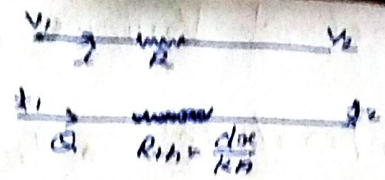
$$\text{Current (I)} = \frac{\text{Potential difference (dV)}}{\text{Electrical resistance (R)}}$$

The heat flow equation (Fourier's equation) is -

$$Q = -K \cdot A \frac{dt}{dx}$$

This equation can be written as -

$$Q = \frac{dQ}{\left(\frac{dx}{KA}\right)}$$



The quantity  $\frac{dx}{KA}$  is called thermal contact resistance ( $R_{th}$ )

$$R_{th} = \frac{dx}{KA}$$

### # Newton's law of Cooling -

"Rate of heat transfer is directly proportional to temp. difference b/w surface and adjacent fluid & area exposed to heat transfer"

$$Q \propto (t_s - t_f)$$

$$Q \propto A$$

$$Q \propto A(t_s - t_f)$$

$$Q = h \cdot A \cdot (t_s - t_f)$$

Where -

$Q$  = Rate of heat transfer.

$A$  = Area exposed to heat transfer.

$t_s$  = Surface temp.

$t_f$  = fluid temp.

$h$  = Heat transfer Coefficient.

Ques-1 - Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick and, whose one face is maintained at 350°C and the other face at 50°C. Take calculate thermal conductivity of copper as 370 W/m°C.

Sol<sup>n</sup> - given data -

(thickness)  $dx = 45 \text{ mm} = \frac{45 \times 1000}{1000} =$

$$dx = 0.045 \text{ m}$$

$$T_1 = 350^\circ\text{C}$$

$$T_2 = 50^\circ\text{C}$$

Temp diff.  $dt = t_1 - t_2$

$$dt = 350 - 50$$

$$dt = 300^\circ\text{C}$$

(Thermal Conductivity)  $K = 370 \text{ W/m}^\circ\text{C}$

$$Q = ?$$

$$Q = K \cdot A \frac{dt}{dx}$$

$$q = K \frac{dt}{dx}$$

Rate of heat transfer per unit area.

$$= 370 \times \frac{300}{0.045}$$

$$= 2466666.67 \text{ kW/m}^2$$

$$\left[ \begin{array}{l} \text{Rate of heat transfer per} \\ \text{unit area} \\ q = \frac{Q}{A} \\ q = K \cdot A \cdot \frac{dt}{dx} \cdot \frac{1}{A} \\ (q = K \frac{dt}{dx}) \end{array} \right]$$

Ques-2: A plate wall is 150 mm thick and its wall area is 4.5 m<sup>2</sup>. if conductivity is 9.35 W/m°C and surface temp. are 150°C and 45°C, determine.

① Heat flow across the plate wall. (Q)

② Temp. gradient in the flow direction. ( $\frac{dt}{dx}$ )

Sol<sup>n</sup> -  $dx = 150 \text{ mm}$   
 $= \frac{150}{1000}$

$dx = 0.15 \text{ m}$

$A = 4.5 \text{ m}^2$

$t_1 = 150^\circ\text{C}$

$t_2 = 45^\circ\text{C}$

(Thermal Conductivity)  $k = 9.35 \text{ W/m}^\circ\text{C}$

Temp. diff.  $dt = t_1 - t_2$

$= 150 - 45$

$dt = 105^\circ\text{C}$

$Q = k \cdot A \frac{dt}{dx}$

$= 9.35 \times 4.5 \times \frac{105}{0.15}$

$= 42.075 \times 700$

$Q = 29452.5 \text{ Watt}$  Ans

ii) Temp. gradient -

$T_g = \frac{dt}{dx}$

$T_g = \frac{-105}{0.15}$

$T_g = 700^\circ\text{C/m}$  Ans



Q3. The following data relates to an oven —

5

Thickness of side wall of oven = 82.5 mm

Thermal conductivity of wall insulation = 0.044 W/m°C

Temp. dissipated by electrical coil within the oven = 40.5 W

Temp on inside of the wall = 175°C

Determine the area of wall surface, perpendicular to heat flow so that temp. on the other side of the wall does not exceed 75°C.

Given data —

$$dx = 82.5 \text{ mm}$$

$$dx = \frac{82.5}{1000}$$

$$dx = 0.0825 \text{ m}$$

$$K = 0.044 \text{ W/m}^\circ\text{C}$$

$$T_1 = 175^\circ\text{C}$$

$$T_2 = 75^\circ\text{C}$$

$$Q = 40.5 \text{ W}$$

$$A = ?$$

$$Q = K \cdot A \frac{dt}{dx}$$

$$dt = T_1 - T_2$$

$$dt = 175 - 75$$

$$dt = 100^\circ\text{C}$$

$$40.5 = 0.044 \times A \times \frac{100}{0.0825}$$

$$A = \frac{40.5 \times 0.0825}{0.044 \times 100}$$

$$= \frac{3.341}{4.4}$$

$$A = 0.759 \text{ m}^2$$

Que-2. A flat plate  $1\text{m} \times 1.5\text{m}$  is maintained at  $300^\circ\text{C}$ . Air  $20^\circ\text{C}$  blows over the plate. If the convective heat transfer coefficient is  $20\text{W}/\text{m}^2\text{C}$ , calculate the rate of heat transfer.

Given data:

$$A = 1 \times 1.5$$

$$A = 1.5\text{m}^2$$

(Surface temp)

$$T_s = 300^\circ\text{C}$$

(fluid temp)

$$T_f = 20^\circ\text{C}$$

$$h = 20\text{W}/\text{m}^2\text{C}$$

$$Q = h \cdot A (T_s - t_f)$$

$$Q = 20 \times 1.5 (300 - 20)$$

$$Q = 30 \times 280$$

$$Q = 8400\text{ watt}$$

$$Q = 8.4\text{ kW}$$

Ans

# Energy Balance

Control  
Volume

Rate of Energy in - Energy out + Energy generated = Rate of Net Energy Stored

$$Q_{x-} - Q_{x+dx} + q_g dv = \rho c \frac{dt}{dt}$$

$$Q_{y-} - Q_{y+dy}$$

$$Q_{z-} - Q_{z+dz}$$

## # General Heat Conduction equation in Cartesian Coordinates

Consider an elemental volume having the coordinates  $(x, y, z)$  for heat conduction analysis

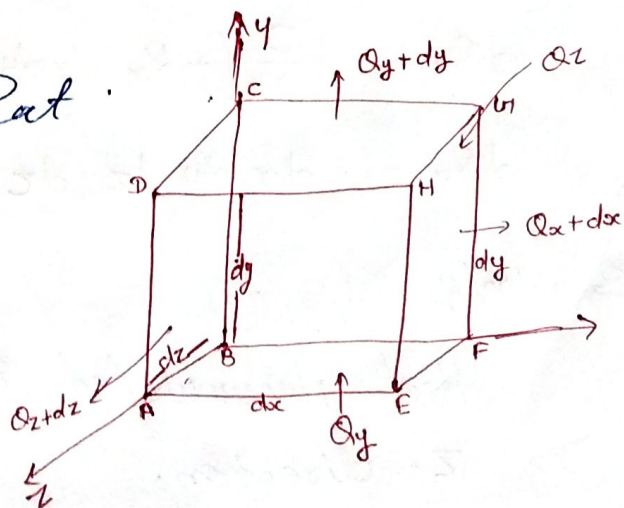
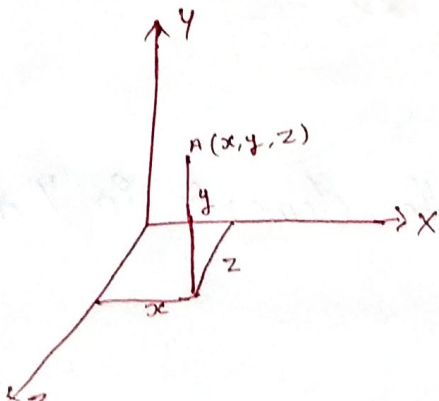
Let.

$q_g$  = Heat generation per meter cube

$K_x, K_y, K_z$  = Thermal conductivities along  $x, y$  and  $z$  axes respectively

$\rho$  = Density

$c$  = Specific heat



Quantity of heat flowing into the element from left face ABCD during the time interval  $d\tau$  in X-direction (heat influx)

$$Q_x = -K_x \cdot A \cdot \frac{\partial t}{\partial x} d\tau$$

$$Q_x = -K_x \cdot dz \cdot dy \cdot \frac{\partial t}{\partial x} \cdot d\tau$$

Quantity of heat flowing out the element from the right face EFGH during the time interval  $d\tau$ .

Heat efflux

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x} (Q_x) dx \quad (\text{Taylor's Series})$$

Heat accumulated in the element in X-direction

$$dQ_x = Q_x - Q_{x+dx}$$

$$dQ_x = Q_x - \left[ Q_x + \frac{\partial}{\partial x} (Q_x) dx \right]$$

$$dQ_x = - \frac{\partial}{\partial x} (Q_x) dx$$

$$dQ_x = + \frac{\partial}{\partial x} \left[ + K_x \cdot dz \cdot dy \cdot \frac{\partial t}{\partial x} \cdot d\tau \right] dx$$

$$dQ_x = dx \cdot dy \cdot dz \cdot d\tau \cdot \frac{\partial}{\partial x} \left[ K_x \cdot \frac{\partial t}{\partial x} \right]$$

Similarly

Heat Accumulated in the element in Y and Z-direction.

$$dQ_y = dx dy dz dz \frac{\partial}{\partial y} \left[ K_y \cdot \frac{\partial t}{\partial y} \right]$$

$$dQ_z = dx dy dz dz \frac{\partial}{\partial z} \left[ K_z \cdot \frac{\partial t}{\partial z} \right]$$

Net heat accumulated in the element =  $dQ_x + dQ_y + dQ_z$

$$dx dy dz dz \frac{\partial}{\partial x} \left[ K_x \cdot \frac{\partial t}{\partial x} \right] + dx dy dz dz \frac{\partial}{\partial y} \left[ K_y \cdot \frac{\partial t}{\partial y} \right] + dx dy dz dz \frac{\partial}{\partial z} \left[ K_z \cdot \frac{\partial t}{\partial z} \right]$$

$$= dx dy dz dz \left[ \frac{\partial}{\partial x} \left( K_x \cdot \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \cdot \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \cdot \frac{\partial t}{\partial z} \right) \right]$$

Total heat generated in the element -

$$Q_g = q_g \cdot dx dy dz dz$$

$$Q_g = q_g \times \text{Volume}$$

Energy balance:-

Net heat accumulated in the element + Energy generated = total energy accumulated in the element.

$$dx dy dz dz \left[ \frac{\partial}{\partial x} \left( K_x \cdot \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \cdot \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \cdot \frac{\partial t}{\partial z} \right) \right] + q_g \cdot dx dy dz dz$$

$$= \rho \cdot c \cdot dx dy dz \cdot \frac{\partial t}{\partial z} \cdot dz$$

$$\frac{\partial}{\partial x} \left( K_x \cdot \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \cdot \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \cdot \frac{\partial t}{\partial z} \right) + q_g = \rho \cdot c \cdot \frac{\partial t}{\partial z}$$

(This is general heat conduction eq<sup>n</sup> Cartesian coordinates //)

Vector form -

$$\nabla(K \cdot \nabla t) + q_g = \rho c \frac{\partial t}{\partial \tau}$$

where -

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

for iso Homogeneous and isotropic material.

$$K_x = K_y = K_z = K$$

$$\frac{\partial}{\partial x} (K \cdot \frac{\partial t}{\partial x}) + \frac{\partial}{\partial y} (K \cdot \frac{\partial t}{\partial y}) + \frac{\partial}{\partial z} (K \cdot \frac{\partial t}{\partial z}) + q_g = \rho c \frac{\partial t}{\partial \tau}$$

$$K \left[ \frac{\partial}{\partial x} \left( \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial t}{\partial z} \right) \right] + q_g = \rho c \frac{\partial t}{\partial \tau}$$

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{K} = \frac{\rho c}{K} \frac{\partial t}{\partial \tau}$$

Proved

Other form -

A- Fourier's equation :- (No Heat generation)

$$q_g = 0$$

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{\rho c}{K} \cdot \frac{\partial t}{\partial \tau}$$

or

$$\nabla^2 t = \frac{\rho c}{K} \cdot \frac{\partial t}{\partial \tau}$$

$$\text{where - } \left\{ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\}$$

Poisson's equation :- (Steady State) ( $t = \text{const}$ ) ②

$$\frac{\partial t}{\partial z} = 0$$

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

c) Laplace equation :- (No heat generation & Steady State Condition)

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or

$$\nabla^2 t = 0$$

# # General Heat Conduction eq<sup>n</sup> in Cylindrical Coordinates \*

Net heat accumulated in the element + Total heat generated

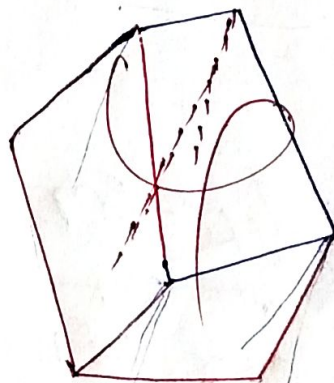
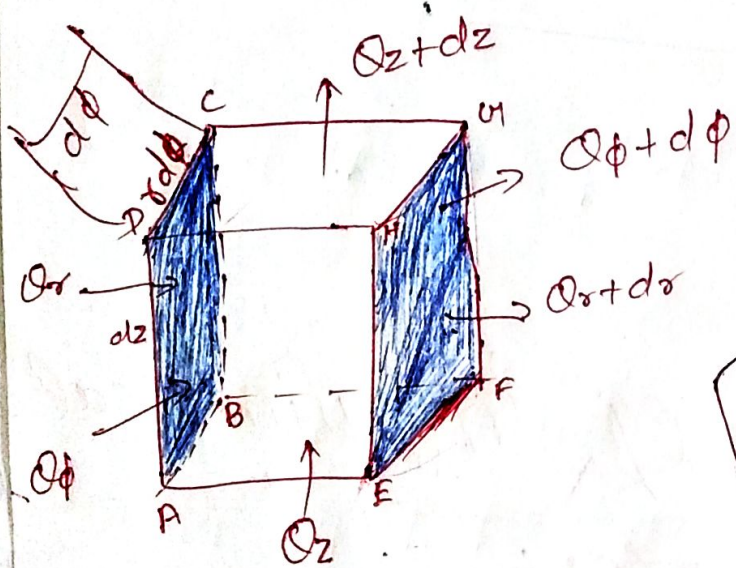
(r-dir. +  $\phi$ -dir. + z-dir.) xyz

$Q_r = Q_r + dr$     $Q_\phi = Q_\phi - d\phi$     $Q_z = Q_z + dz$

$Q_g = q_g \cdot \text{Vol.} \times dz$

= Total Heat Stored in the element.

$$m c \frac{\partial t}{\partial z} \cdot dz$$



Confusion point -

Ex -

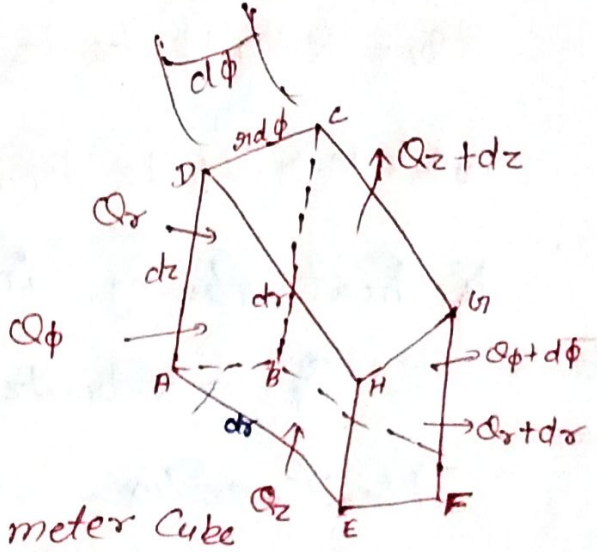
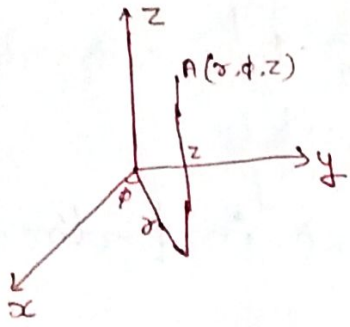
$$Q_r = -K \cdot A_{\text{area}} \cdot \frac{\partial t}{\text{(distance travelled)}} \cdot dz$$

$$Q_r + dr = Q_r + \frac{\partial}{\text{(distance travelled)}} (Q_r) \times (\text{distance travelled})$$



Assumptions

Consider an element Volume having the coordinates  $(r, \phi, z)$  for three dimensional heat conduction analysis -



- let -  $q_g$  = Heat generation per meter cube
- $\rho$  = density
- $dV = r \cdot d\phi \cdot dr \cdot dz$
- $C$  = Specific heat

The heat flowing in the element in  $r$ -direction

Heat influx - ABCD

$$Q_r = -K \cdot r \cdot d\phi \cdot dz \cdot \frac{\partial t}{\partial r} \cdot dr$$

Heat efflux

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} (Q_r) dr$$

Heat accumulated in the element in  $r$ -direction

$$dQ_r = Q_r - Q_{r+dr}$$

$$dQ_r = Q_r - [Q_r + \frac{\partial}{\partial r} (Q_r) dr]$$

$$dQ_r = -\frac{\partial}{\partial r} (Q_r) dr$$

$$dQ_r = -\frac{\partial}{\partial r} \left[ K \cdot r d\phi \cdot dz \cdot \frac{\partial t}{\partial r} \cdot dz \right] dr$$

$$dQ_r = K dr d\phi \cdot dz \cdot dz \cdot \frac{\partial}{\partial r} \left[ r \cdot \frac{\partial t}{\partial r} \right]$$

$$dQ_r = K \cdot dr d\phi \cdot dz \cdot dz \cdot \left[ r \cdot \frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r} \cdot 1 \right] \quad \because dv = r d\phi \cdot dz \cdot dr$$

$$dQ_r = K \cdot r \cdot d\phi \cdot dz \cdot dr \cdot dz \cdot \left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} \right]$$

the heat flowing in the element in  $\phi$ -direction

$$Q_\phi = -K \cdot dr \cdot dz \cdot \frac{\partial t}{r \cdot \partial \phi} \cdot dz$$

Heat efflux -

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{\partial \phi} (Q_\phi) \cdot r d\phi$$

Heat accumulated in the element in  $\phi$ -direction.

$$dQ_\phi = Q_\phi - Q_{\phi+d\phi}$$

$$dQ_\phi = Q_\phi - \left[ Q_\phi + \frac{\partial}{\partial \phi} (Q_\phi) r d\phi \right]$$

$$dQ_\phi = -\frac{\partial}{\partial \phi} \left[ -K \cdot dr \cdot dz \cdot \frac{\partial t}{r \cdot \partial \phi} \cdot dz \right] \cdot r d\phi$$

$$dQ_\phi = \frac{1}{r} \cdot K \cdot dr \cdot dz \cdot d\phi \cdot dz \cdot \frac{\partial^2 t}{\partial \phi^2}$$

$$dQ_\phi = \frac{1}{r^2} \cdot K \cdot r d\phi \cdot dr \cdot dz \cdot dz \cdot \frac{\partial^2 t}{\partial \phi^2}$$

Heat flowing in the element in z-direction

Heat influx

$$Q_z = -K \cdot r d\phi \cdot dr \cdot \frac{\partial t}{\partial z} \cdot dz$$

Heat efflux -

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z} (Q_z) dz$$

Heat accumulated in the element in z-direction.

$$dQ_z = Q_z - Q_{z+dz}$$

$$dQ_z = - \frac{\partial}{\partial z} (-K \cdot r \cdot d\phi \cdot dr \cdot \frac{\partial t}{\partial z} \cdot dz) dz$$

$$dQ_z = K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \frac{\partial^2 t}{\partial z^2}$$

Net Heat accumulated in the element

$$= dQ_r + dQ_\phi + dQ_z$$

$$= K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right] + \frac{1}{r^2} \cdot K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \frac{\partial^2 t}{\partial \phi^2} + K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \frac{\partial^2 t}{\partial z^2}$$

$$= K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} \right]$$

(Net Heat)

Total Heat generated in the element

$$Q_g = q_g \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz$$

Energy Balance

Net Heat accumulated in the element + Total Heat generated = Total Heat accumulated in the element.

$$K \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz \left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} \right] + q_g \cdot r \cdot d\phi \cdot dr \cdot dz \cdot dz = \rho \cdot r \cdot d\phi \cdot dr \cdot dz \cdot c \cdot \frac{\partial t}{\partial z} \cdot dz$$

$$\left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{K} = \frac{\rho c}{K} \cdot \frac{\partial t}{\partial z} \right]$$

This is the Heat Conduction eq in Cylindrical Coordinates.

# # Heat Conduction through a plane wall

## Assumptions

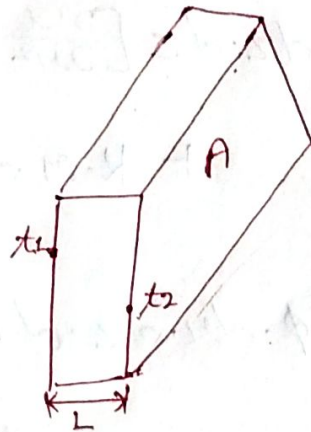
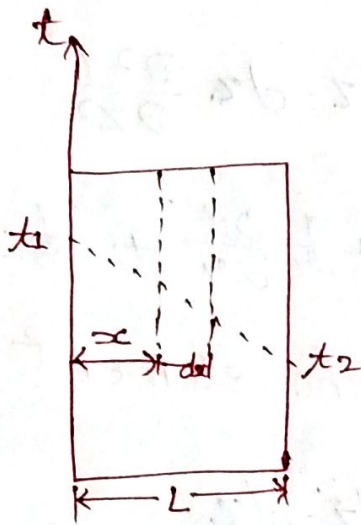
1. Consider a plane wall of homogeneous material.
2. Uniform thermal conductivity.
3. Heat flow in only  $x$ -direction.

$L$  = Thickness of the wall.

$A$  = Cross-sectional area of the wall.

$t_1$  = Temp. of left face of wall.

$t_2$  = Temp. of Right face of wall.



The general Heat Conduction eq<sup>n</sup> for Cartesian Coordinates is given by -

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{K} = \frac{\rho c}{K} \cdot \frac{\partial t}{\partial \tau} \quad \text{--- (1)}$$

for no internal heat generation, Steady State Condition and heat flow in  $x$ -direction.

$$\frac{q_g}{K} = 0 \quad \frac{\partial t}{\partial \tau} = 0$$

$$\frac{\partial^2 t}{\partial y^2} = 0 \quad \frac{\partial^2 t}{\partial z^2} = 0$$

for equation - (1)

$$\frac{\partial^2 t}{\partial x^2} = 0$$

By integrating this eq<sup>n</sup> twice, then we get.

$$\frac{\partial t}{\partial x} = C_1$$

$$t = C_1 x + C_2 \quad \text{--- (2)}$$

Applying Boundary Conditions-

$$\left\{ \begin{array}{l} x=0 \\ t=t_1 \end{array} \right\} \quad \left\{ \begin{array}{l} x=L \\ t=t_2 \end{array} \right\}$$

$$t_1 = C_1 \times 0 + C_2$$

$$\boxed{t_1 = C_2}$$

$$t_2 = C_1 \times L + t_1$$

$$\boxed{\frac{t_2 - t_1}{L} = C_1}$$

$$\boxed{t = \frac{t_2 - t_1}{L} \cdot x + t_1}$$

(x<sup>1</sup>)  
(x<sup>2</sup>)x

this eq<sup>n</sup>. indicates that temp. distribution across the wall is linear.

• Heat flux or Rate of heat transfer

$$Q = -K \cdot A \frac{dt}{dx}$$

$$Q = -K \cdot A \frac{d}{dx} \left[ \frac{t_2 - t_1}{L} \cdot x + t_1 \right]$$

$$Q = -K \cdot A \frac{(t_2 - t_1)}{L}$$

$$\boxed{Q = \frac{K \cdot A (t_1 - t_2)}{L}}$$

or

$$\boxed{Q = \frac{t_1 - t_2}{\frac{L}{KA}}}$$

where  $\frac{L}{KA} = R_{th}$  (thermal Resistance)